

## *Zaznavanje oscilatornih karakteristik elektroenergetskega sistema v realnem času*

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**Povzetek** – Težave s stabilnostjo so inherentne za elektroenergetske sisteme zaradi njihove nelinearne in dinamične narave. Izzivi stabilnosti za majhne motnje, ki so predmet tega prispevka, so opazni, kadar en ali več sinhronskih generatorjev niha drug z drugim. Slabo dušena nihanja lahko brez ustreznih protiukrepov vodijo do nestabilnosti sistema, kar se lahko konča z njegovim razpadom. Modalna analiza se je sicer izkazala kot ustrezen pristop za preučevanje slabo dušenih nihanj s simulacijami na računalniških modelih. Po drugi strani lahko stabilnost sistema ocenimo v realnem času s pomočjo meritev kazalcev električnih veličin. Pri obratovanju realnega sistema pa zaenkrat tudi v najbolj optimističnih razmerah ne moremo pričakovati tolikšnega vpogleda v obratovalno stanje sistema kot je mogoč z modalno analizo na računalniških modelih. V prispevku je opisana metoda, ki omogoča povezavo med oceno stabilnosti s sprotimi meritvami in rezultati računalniškega modela z uporabo analize glavnih komponent. Metoda je preizkušena s pomočjo orodja DigSILENT Power Factory, in sicer na 39 vozliščnem modelu. Na osnovi modela je bila sestavljena podatkovna baza tako z rezultati modalne analize kot tudi z meritvami nihanj iz časovnih potekov merjenih veličin. Obdelava podatkov je izvedena v programskem okolju Matlab.

**Ključne besede:** stabilnost za majhne motnje, analiza glavnih komponent, spektralna analiza, nihanja, Power Factory, modalna analiza, spoznavnost načina

### *Real-time identification of oscillatory characteristics of power systems*

**Abstract** – Stability problems are inherent to power systems due to their non-linear and dynamic nature. The stability challenges for small disturbances, which are the subject of this paper, are noticeable when one or more synchronous generators oscillate relative to each other. Poorly damped oscillations, without adequate countermeasures, can lead to instability of the system, which can lead to its collapse. Modal analysis proved to be a suitable approach for the study of poorly damped oscillations with simulations on computer models. A different approach is implemented in real-time, made possible by utilizing phasor measurements of electrical quantities. However, when operating a real system, even under the most optimistic conditions, we cannot expect as much insight into the operating state of the system as it is possible with modal analysis on computer models. This paper describes a method that allows the connection between real-time measurements and the modal analysis results of a computer model using principal component analysis. The method is tested using the DigSILENT Power Factory software on a 39-node model. Based on the model, a database was compiled with the results of the modal analysis as well as with the timely fluctuations of the measured quantities. The data processing is performed in the Matlab software environment.

**Keywords:** small-signal stability, principal components analysis, spectral analysis, Power Factory, power system oscillations, modal analysis, mode shape

## 1 INTRODUCTION

In recent years, power systems have been subjected to many novelties in their operation. Emerging technologies in the areas of renewable energy sources, power electronic devices, distributed generation etc. have made a substantial impact on power systems. For example, the highly variable output power of most renewable energy sources can have a considerable effect on the dynamic behavior of power systems, leading to power swings and less synchronizing coupling [1]. It is safe to say that the issue at hand causes significant problems in the secure and reliable operation of the power system i.e., its ability to withstand disturbances. The terms secure and reliable can refer to many aspects in the operation, all of which can be eventually linked to the stability of power systems. From the many different types of stability, in this paper, we direct our focus to small signal stability, or the system's ability to maintain synchronism under small disturbances [2].

The most utilized tool for assessing small-signal stability is modal analysis [3]. The method is of mathematical nature and utilizes the linearization of the differential equations representing the system and solving for its eigenvalues and eigenvectors. This provides the information about the small signal dynamics of the system. Each eigenvalue contains the information on the frequency and damping of the oscillations that appear in the system (so-called *mode*). For each eigenvalue, we have the left and right eigenvector. The latter is usually referred to as *mode shape* and shows the activity of each generator in that specific mode. The problem with modal analysis is that it deals with a mathematical model of a particular operating scenario of the system, thus it is unable to assess the small signal dynamics of real-time, evolving systems, whose exact operating characteristics can hardly be considered as known.

An "improvement" on modal analysis came with real-time measurements, made possible by the introduction of Phasor Measuring Units (PMUs) in power systems. The first methods concentrated on estimating the frequency and damping of oscillations, while later, mode shape estimation methods became a useful reality. Assessing the mode shape has the potential to aid in the mitigation of power system oscillations by providing information for taking adequate control actions [4].

With the frequency, damping and mode shape, we already have a good idea of what is happening in the power system in the terms of its stability. In this paper, our aim is to further improve our knowledge about the stability in our system by recognition of modes with similar mode shapes in our database made of modal analysis results. We make that possible utilizing Principal Components Analysis on our database, which in turn reduces its dimensionality and provides means for comparing mode shapes and visualizing modes through their mode shapes. Firstly, let us provide some theoretical background on the meaning of small signal stability and modal analysis as the prime assessor of small signal stability.

## 2 SMALL SIGNAL STABILITY

As mentioned in the introduction, small signal stability refers to the power system's ability to stay in synchronism when subjected to small disturbances. Further definitions and categorizations of power systems stability can be found in [5]. The disturbance is considered small if we can linearize the differential equations defining the system for the purpose of our analysis. For example, let us consider the movement of the rotor of the synchronous generator according to the swing equation:

$$\frac{2 \cdot H}{\omega_0} \cdot \frac{d^2\theta}{dt^2} + D \cdot \frac{d\theta}{dt} = P_m - P_e(\theta) \quad (1)$$

where,  $\omega_0$  is the nominal angular frequency,  $H$  is the constant of inertia of the generator,  $D$  is the damping constant of the generator,  $\theta$  is the rotor angle,  $P_m$  is the mechanical power of the turbine driving the shaft and  $P_e(\theta)$  is the electrical power of the generator, as a function of the rotor angle.

During stable operation, the mechanical power of the turbine  $P_m$  is equal to the electrical power that is fed into the grid  $P_e$ , thus the generator is in equilibrium, and  $\theta$  is held constant. When a small disturbance happens in the grid, the rotor angle of the generator deviates from its initial position by  $\Delta\theta$  and takes the value of, for example  $\theta_1$ , as shown in Figure 1-a.  $P_e(\theta)$  now takes the value of  $P_e(\theta_1)$ , while  $P_m$  stays the same, making the right-hand side of

the equation not equal to zero. Solving (1) for the rotor angle with consideration of the inequality on the right-hand side, one would yield a solution for the rotor angle in the form of:

$$\theta(t) = \theta_0 + K \cdot e^{\sigma \cdot t} \cdot \cos(\omega_n \cdot t) \quad (2)$$

In (2),  $K$  is determined by the severity of the disturbance,  $\sigma$  is the absolute damping and  $\omega_n$  is the natural (damped) angular frequency of the oscillations of the rotor angle. Figure 1-b, shows the time response of the rotor angle  $\theta$  during a small disturbance, which is an oscillation around the values  $\theta_1$  and  $\theta_2$  (no damping).

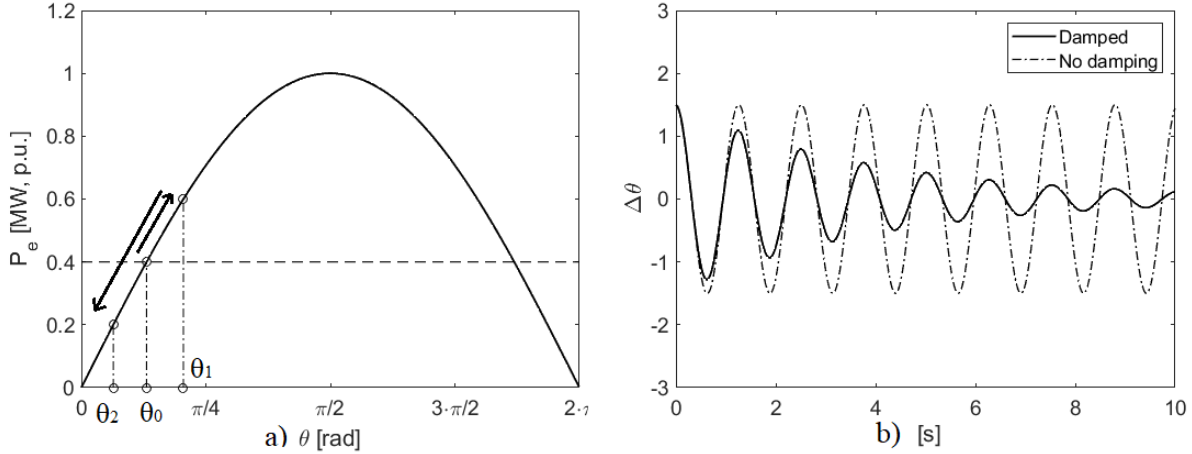


Figure 1. Oscillations of the rotor angle

During a disturbance like this, the movement of the rotor is a consequence of a resulting electromagnetic torque which can be separated into two components as follows [6]:

- *synchronizing torque*, which is in phase with the rotor angle deviation, and
- *damping torque*, which is in phase with the rotor speed deviation.

The idea behind small signal stability is that, during a disturbance, the generator has enough synchronizing and damping torque so that the oscillations quickly dissipate and therefore cannot be a cause for further worsening of system stability.

## 2.1. Modal analysis

In real power systems, the problem is much more complicated as opposed to when dealing with just one generator. One or many different generators tend to swing together or relative to each other, making the calculations like the previous practically impossible. The time response is almost never as simple as shown in Figure 1. Usually, the form of the oscillogram is a sum of multiple sine wave oscillations, each with their own damping and frequency. The key feature of modal analysis is to find all those oscillations (modes) in the system and quantify their presence in each of the generators. Modal analysis assumes the linearized dynamical system with zero input:

$$\dot{x} = A \cdot x \quad (3)$$

The solution of such system in the Laplace domain is nicely conveyed in [7] and is represented by:

$$x(s) = \sum_{j=1}^n \frac{q_j^T \cdot x(0)}{s - \lambda_j} \cdot p_j \quad (4)$$

Equation (4) shows that the response of the system, in the case of a small disturbance, depends on the initial conditions  $x(0)$ ,  $q_j^T$ ,  $p_j$ , and  $\lambda_j$ . The values  $\lambda_j$  are eigenvalues (in the form of  $\sigma_j \pm j \cdot \omega_j$ ) of the state matrix  $A_{n \times n}$ , while  $q_j^T$  and  $p_j$ , are the left and right eigenvector of the adequate eigenvalue, respectively. It is important to note that  $q_j^T$  is a row vector,  $x(0)$  is a column vector and  $p_j$  is a column vector, all with  $n$  values. This eventually means that the information of how much a certain generator  $G_i$  oscillates at the mode  $\lambda_j$  i.e. with damping  $\sigma_j$  and frequency  $\omega_j$ , depends on the corresponding value  $p_{ij}$  in  $p_j$ . The eigenvector  $p_j$  is called the *mode shape* for the eigenvalue

$\lambda_j$  and gives insights into the activity of each state variable in  $x$  for that mode  $\lambda_j$ . Often, the values in  $\mathbf{p}_j$  are of complex nature with a given magnitude and phase. Figure 2-a shows the time response of the rotor angle of three generators during a small disturbance which excites an oscillation of 0.637 Hz. Figure 2-b shows the mode shape of that time response where we can distinguish between the magnitude and the relative phase shift i.e. the oscillation of G1 against G2 and G3.

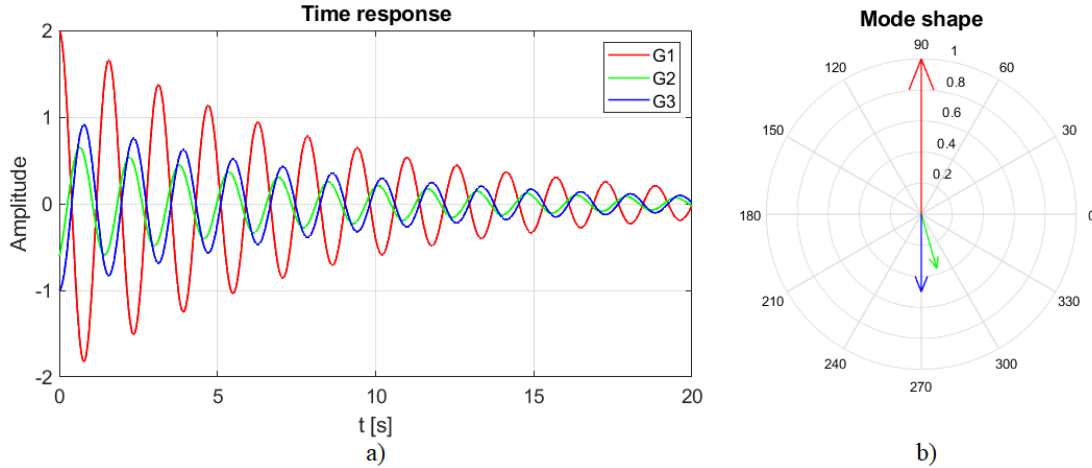


Figure 2. Mode shape of a 0.637 Hz oscillation for three generators

### 3 MODE SHAPE ESTIMATION AND RECOGNITION

#### 3.1. Mode shape estimation

For the purpose of this paper, we rely on the assessment of the mode shape of oscillation from the ambient response of the system. The ambient response is the product of small disturbances which excite certain modes in the system, thus we research on the reverse possibility of obtaining the mode shape/mode of the oscillation through ambient measurements. Many different methods have been developed for that purpose; we refer to the findings in [4] for a detailed comparison of available methods. In this paper, we estimate the mode shape from ambient measurements utilizing the power spectral density and cross power spectral density method, conveyed in [8]. The method calculates the powers  $\mathcal{S}(\omega)$  of ambient measurements ( $\mathcal{S}$  is a column vector), at each frequency and relates it to the square of the mode shape at that given frequency  $\mathbf{p}^2$ . Understandably, the frequencies at which exact modes of oscillations appear, would have much higher power than a random selected frequency, thus the relation develops into:

$$\mathcal{S}(\omega_j) \sim \mathbf{p}_j^2 \quad (5)$$

At the end, we just scale the values of the mode shape so that the eigenvector has unity length. For the angle of the mode shape, we calculate the cross-power spectral density of all measurements in relation to the one with the highest relative mode shape magnitude obtained in the previous step. Suppose the  $k$ -th element of  $\mathbf{p}_j$  has the highest value for the mode shape magnitude, then the relative angle of the mode shape of the measurement  $m$ , would be:

$$\angle \mathbf{p}_{mj} = \angle \mathcal{S}_{km}(\omega_j) \quad (6)$$

With this step we have calculated the mode shape i.e. its magnitude and phase, from the ambient measurements of our power system. This was the prerequisite for continuing to the next step which is the recognition of the mode shape in the database.

#### 3.2. Mode shape recognition

The recognition of the estimated mode shape and its comparison with a similar mode shape in the database is done using principal components. Principal Component Analysis, or PCA, is a statistical technique used to reduce the dimensionality of large datasets by identifying the most important patterns and relationships in the data. It does this by identifying the principal components of the data, which are the directions that explain the most variation,

and projecting the data onto these components to create a lower-dimensional representation of that same data. Reference [9] and [10] provide insights into data driven trends in power engineering and engineering in general.

Given a dataset  $\mathbf{X}$  with  $n$  observations and  $p$  features, we can represent it as an  $n$  by  $p$  matrix  $\mathbf{X}$ , where each row represents an observation, and each column represents a feature. The goal of PCA is to find a lower-dimensional representation of  $\mathbf{X}$  that captures the most important patterns and relationships in the data. This is done by finding the principal components of  $\mathbf{X}$ , which are the directions that explain the most variation in the data. The first principal component can be found by finding the eigenvector of the *covariance matrix* of  $\mathbf{X}$  that corresponds to the largest eigenvalue. Subsequent principal components can be found by finding the eigenvectors corresponding to the next largest eigenvalues. Once the principal components have been identified,  $\mathbf{X}$  can be projected onto these components to create a lower-dimensional representation of the data. This can be done by multiplying  $\mathbf{X}$  by the matrix of principal components  $\mathbf{W}$ . The resulting matrix will have the same number of observations as  $\mathbf{X}$ , but only as many columns as there are principal components. Equation (7) represents the transformation from the “regular” to the principal components space of the data (from 1000 to only 3 features), also represented in Figure 3. We now can compare the observations with each other using only the 3 features provided with the PCA.

$$PC_{n \times 3} = X_{n \times 1000} \cdot W_{1000 \times 3} \quad (7)$$

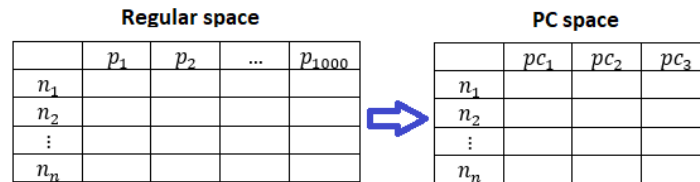


Figure 3. Regular and PC space

#### 4 SIMULATION RESULTS

There are several steps to our concept of identifying the oscillatory characteristics of power systems. These can be divided as follows:

1. database creation and PCA,
2. mode shape estimation from ambient measurements, and
3. mode shape recognition in database.

The validity of the proposed method is tested on the New England 39 bus test system shown in Figure 4. The simulations were performed in the PowerFactory simulation software.

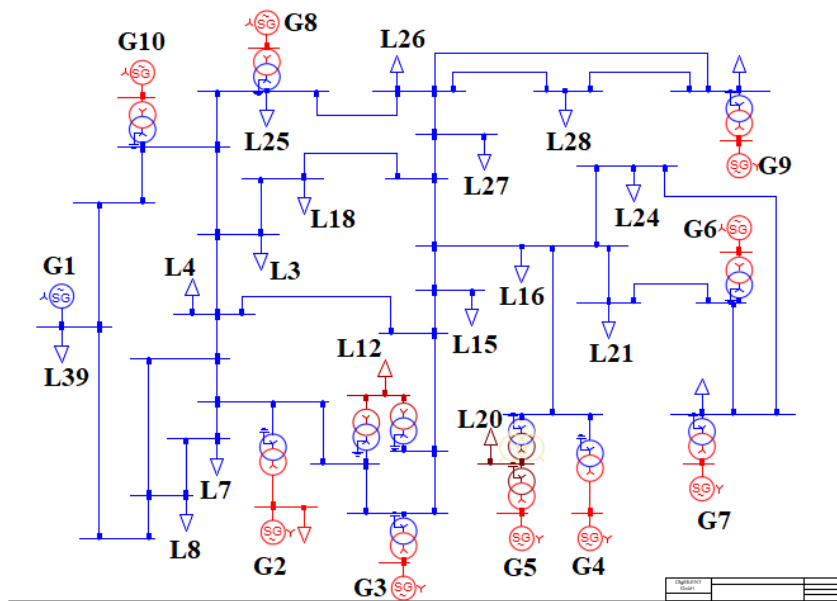


Figure 4. New England 39 bus test system (data can be found in [11])

Firstly, we created our database of five different scenarios of the test system with the idea of obtaining variations in the modal analysis data (difficult with a small system). The scenarios are obtained by changing generator output, changing the topology of the grid and/or the power of the loads. The database is constructed using the mode shapes of the rotor angles of the generators for each calculated mode which has a damping range up to 10%, from each scenario. The values in the mode shape vector are presented in Figure 5 with their cartesian coordinates. Each row in the database presents a mode, while the columns are the real and imaginary values of the mode shape for that mode. After preparing the dataset of mode shapes, the PCA algorithm is applied using the inbuilt function *pca* in Matlab.

mode number	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
4	0.8158	0.0000	0.1741	0.1777	1.0000	0.1507	0.7830	-0.2313	0.8461	0.0222	0.4496	0.0000	0.0285	-0.0384	0.0000	0.0016	-0.0867	0.5018	-0.2040	0.4619
5	0.1623	0.0000	0.1953	1.0000	-0.1174	0.8417	-0.0021	0.3195	0.1976	0.2293	-0.0815	0.0000	0.0368	0.0000	0.1082	-0.0966	0.2003	-0.0798	-0.0007	-0.0377
6	1.0000	0.0000	-0.0504	-0.2774	-0.4078	-0.3247	-0.3314	0.0171	-0.4357	0.0583	0.0000	0.0000	0.0002	-0.0099	-0.0082	-0.0230	-0.0427	-0.0024	0.0049	-0.0542
7	0.9204	0.0000	0.4464	0.6093	1.0000	0.6029	0.9462	0.7299	0.8430	0.7932	0.0978	0.0000	-0.1606	-0.0124	0.0000	-0.0106	-0.0880	0.1274	0.0039	0.0965

Figure 5. Database created with the cartesian coordinates of the mode shapes

Secondly, we needed ambient measurements from which we can estimate the mode shape of the eventual oscillations which appear in the system. Because, to our knowledge, there is no such tool provided with Power Factory, the ambient simulations were created using random small disturbances in the grid. More precisely, we created a random load change (random consumer and random range from -10% to 10%) at each 3 second interval, with the idea that such small disturbances would excite the electromechanical modes in the system. We then observed the relative (in relation to the reference rotor angle) rotor angles of each generator for a simulation lasting 10 minutes and obtained the graph in Figure 6.

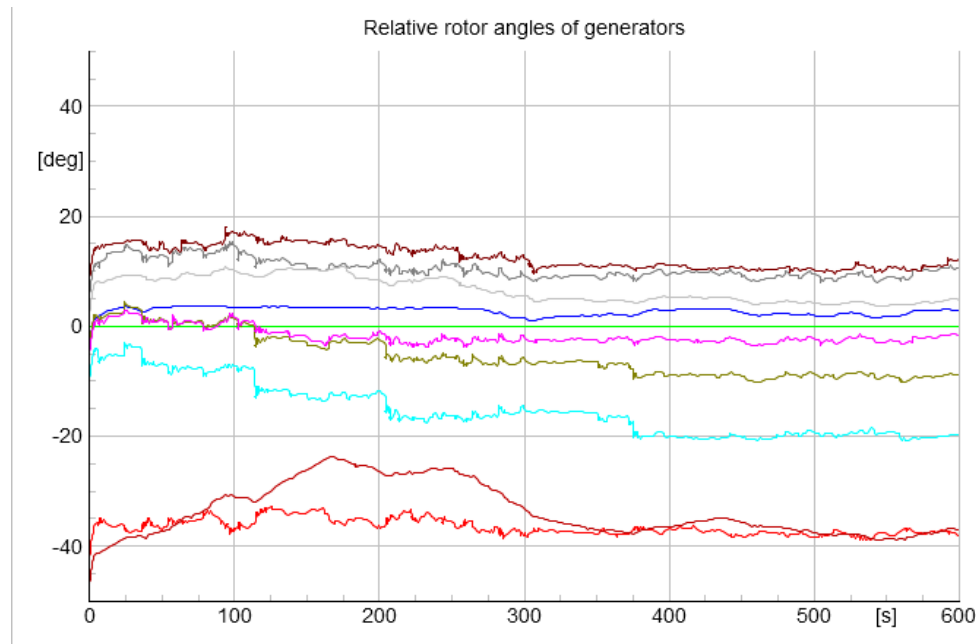


Figure 6. Measured rotor angles for the duration of the simulation

The measurements show how the disturbances introduced in our grid excite a response of the rotor angles. For real-life implementation, we would have to take other measurable quantities like bus frequency, voltage angle etc. having in mind that rotor angles cannot be measured. Understandably, that would mean that the values in the database (the mode shapes) should be for those measurable quantities. From those measurements, we can estimate the mode shape using the power spectral density and cross power spectral density as conveyed in section 3.1. The estimation can be done automatically, or in our case, manually by selecting the frequency at which there is an expressed power range in the spectral analysis as shown in Figure 7. The power spectral density and cross power spectral density for the measured rotor angles are calculated using the *pwelch* and *cpsd* functions in Matlab. Here, we can mention that the method which estimates the mode shape is irrelevant for the whole concept i.e. the estimation and the recognition parts of the concept are separate and not dependent on each other. That means that we can use any other method for mode shape estimation and still yield satisfactory results. Efforts are underway to compare the results of using a different mode shape estimation algorithm, more precisely the frequency domain decomposition method conveyed in [12].

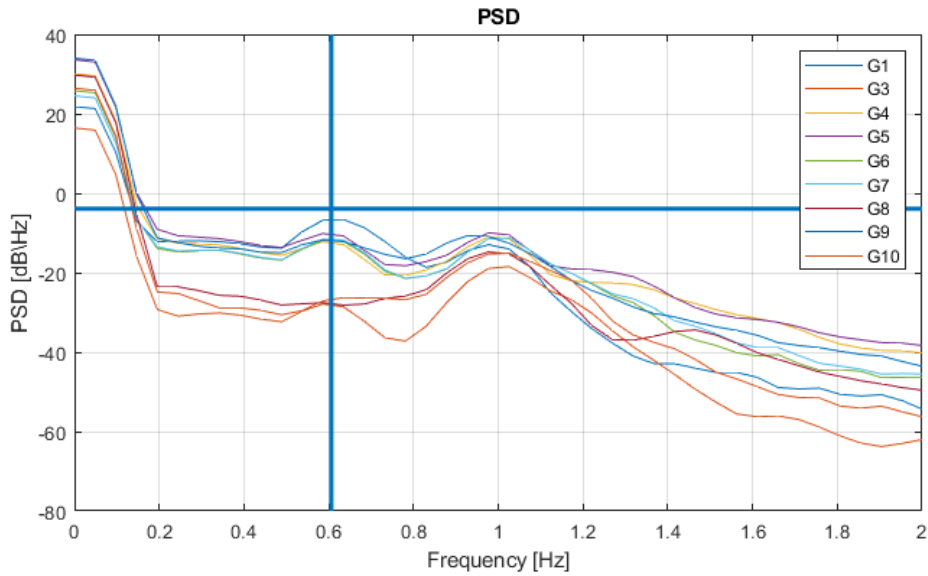


Figure 7. Power spectral density of rotor angle measurements (one selected frequency)

The third step is to recognize the most similar modes in our database i.e. find a mode shape/mode shapes with similar characteristics. The idea is to find more than one similar mode shape because of the errors introduced with the estimation. This would mean that the result would be in a form of a cluster of modes, which have the most similar mode shapes to the one which was estimated using the ambient measurements. This is achieved using the calculated coefficients in the first step and transforming the estimated mode shape from the “regular” to the PC space. Furthermore, we used k-means clustering on the principal components to obtain 9 clusters of modes (the number of modes in one scenario). K-means is a clustering method which partitions the data into the desired number of clusters, where each point belongs to the cluster with the closest mean. The idea is that even if the method falls short in finding the closest scenario in the database, the estimated mode shape will almost certainly correctly fit to the closest cluster of mode shapes. Figure 8, shows the PCA representation of the database and the estimated mode with a red “x”. On the right-hand side of the figure, we can also observe the clustering of the modes, where each cluster is presented with its corresponding centroid with a large, black colored “x”. The mode shapes of the modes coming from the same cluster as the estimated mode shape are also shown in Figure 9.

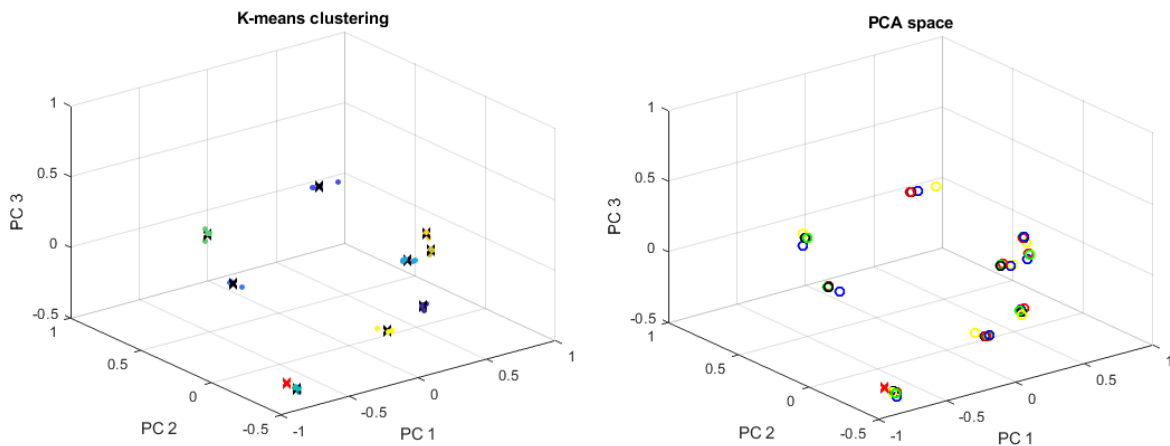


Figure 8. Modes in the PCA space

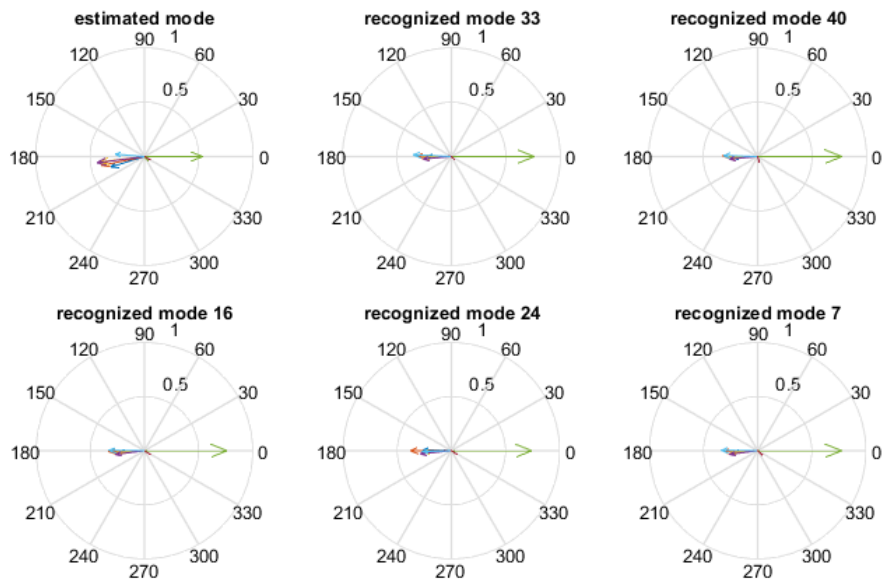


Figure 9. Recognized similar modes

We can draw several conclusions from the observed results. One important drawback, or issue that needs further addressing, is that we cannot be 100% positive about the accuracy of the estimated mode shape. The error introduced with the estimation could have a significant impact to the recognition of the mode shape latter in the process. Future work would eventually need to focus on this matter utilizing the approach conveyed in this paper or any other alternative method for mode shape estimation. On the other hand, concerning PCA as a dimensionality reduction technique, we showed that it can be successfully used in the visualization of modes through their mode shapes. Furthermore, in addition with k-means clustering it provides a useful approach to classifying similar modes together. When it comes to the usefulness of the method as a complete concept, which for future work needs to be validated on real life data, we argue that the real time nature of the method, together with the relative simplicity of the output results, provide important initial information about what is happening in our power system. Based on the results, additional analysis can be directed toward already familiar scenarios and adequate contingency measures can be evoked.

## 5 CONCLUSION

In this paper, we presented a method for real-time identification of the electromechanical characteristics of power systems. The steps to implementing the method started with the creation of the database, comprised of the mode shapes of poorly damped modes of a certain power system. Principal components analysis was conducted on that database, which in turn allowed for the plotting and clustering of similar modes according to their mode shape, which appear in different scenarios of the same system. In addition, we used spectral analysis to estimate a real-time mode shape from ambient measurements which were obtained through a 10 min. simulation where we measured the response of the system during small disturbances. The estimated mode shape shares large similarities with a cluster of modes from the database, which was presented both in the PCA space and in a plot showing the mode shapes in polar coordinates. With the identified frequency of oscillations, the mode shape of the oscillations and the cluster in which the system would most likely appear, the method has the potential to be used for a fast, rough, estimation of the oscillatory characteristics of power systems.<sup>1</sup>

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